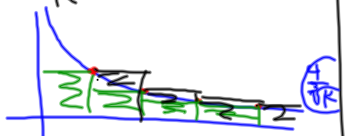
8.3 series / # 1

(a) $\sum_{K=1}^{\infty} \frac{4}{\sqrt[3]{K}}$



$4\int_1^{\infty} (K)^{-1/3}\, dK$

$= 4 \lim_{b\to\infty} \int_1^b K^{-1/3}\, dK$

$= 4 \lim_{b\to\infty} \left(\frac{K^{2/3}}{2/3} \right)\Big|_1^b$

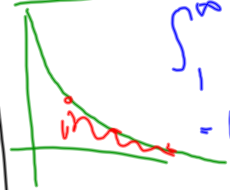
$= 4 \lim_{b\to\infty} \frac{3}{2} b^{2/3} - \frac{3}{2}(1)$

$= \infty$

(b) $\sum_{K=1}^{\infty} K^{-9/10}$

aside

$\lim_{x\to\infty} x^{?}$ converges



$\int_1^{\infty} x^{-9/10}\, dx$

$= \lim_{b\to\infty} \int_1^b x^{-9/10}\, dx$

$= \lim_{b\to\infty} \frac{x^{\frac{1}{10}}}{\frac{1}{10}} \Big|_1^b$

$= \lim_{b\to\infty} 10\, b^{1/10} - 10(1)^{1/10}$

$= \infty$

$10^9 ? 10^{10}$ (= when K=)

$\sum_{K=1}^{\infty} \frac{1}{K^{9/10}}$

$\frac{1}{10^9}\ \frac{1}{K^{9/10}} \geq \frac{1}{K^1}\ \frac{1}{10^{10}}$

$<$

$>$

$<$ $K = 10^{10}$

$=$

$>$

$=$

$=$

$\frac{1}{16} + \cdots \frac{8}{16}$

$\frac{1}{9} + \cdots \frac{1}{16}$

$\sum_{K=1}^{\infty} \frac{1}{K}$ diverges. Harmonic Series.

$\sum_{K=1}^{\infty} \frac{1}{K}$

$1 + \frac{1}{2} + \underbrace{\frac{1}{3} + \frac{1}{4}}_{\text{bigger than } \frac{1}{4}+\frac{1}{4} = \frac{1}{2}} + \underbrace{\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}}_{\text{bigger than } \frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8} = \frac{1}{2}} + \cdots$

Comparison

if $a_n \geq b_n$ & $\sum_{K=1}^{\infty} b_K$ diverges

Then $\sum_{K=1}^{\infty} a_K$ diverges

if $0 \leq a_n \leq b_n$ And $\sum_{K=1}^{\infty} b_K$ Converges

then $\sum_{K=1}^{\infty} a_K$ also converges

Series you can compare to:

if you need a divergent series:

$\sum_{k=1}^{\infty} \frac{1}{k}$ harmonic

$\sum_{k=1}^{\infty} (b)^k$ geometric (when $|b| \geq 1$)

any series where $\lim_{k \to \infty} a_k \neq 0$

if you need a convergent Series

$\sum_{k=1}^{\infty} b^k$ geometric when $|b| < 1$

$\sum_{k=1}^{\infty} \frac{1}{k^p}$ (when $p > 1$) (p-series)

integral

$\int_1^{\infty} x^{-p}\,dx$

$1 - p < 0 \Rightarrow$ Converges

$1 < p \Rightarrow$ Converges

implicit differentiation

$$x^2+2xy+y^2=0$$

diff w/r/t x

$$\frac{dx}{dx}=1 \quad \frac{dy}{dx}=\frac{dy}{dx}$$

Solve for $\frac{dy}{dx}$

L'Hospital's Rule

$$\lim_{x\to ?}\frac{f(x)}{g(x)}=\frac{0}{0} \text{ or } \frac{\infty}{\infty}$$

$$\parallel$$

$$\lim \frac{f'(x)}{g'(x)}$$

Definite integrals = #

= positive (or negative) area

$$\int_a^b f(x)\,dx = \lim \sum_{k=1}^{n} f(x_k)\Delta x_k$$

$$= F(b) - F(a)$$

FTC

where $F(x)$ Any Antiderivative

$$\int_a^{\infty} f(x)\,dx \;/\; \int_{-\infty}^{a} f(x)\,dx$$

OR

$$\int_a^b f(x)\,dx$$

where $f(x)$ has an asymptote (vert) bet. a & b

Volume of

- solids of revolution (disks or washers)

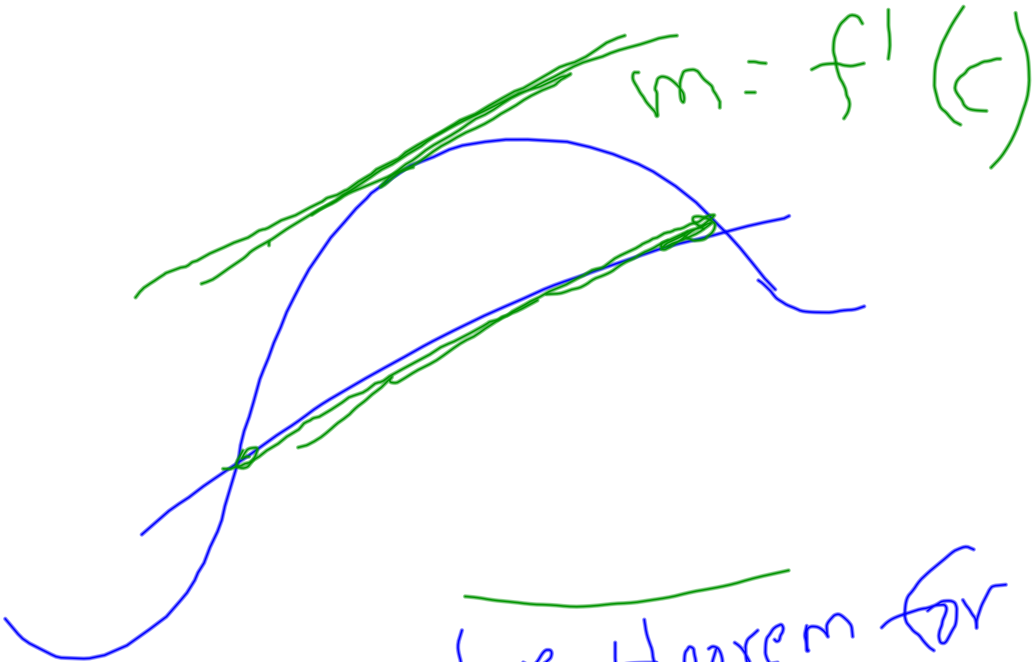
- solids w/ known cross-sectional area

$$V = \int_a^b \text{cross-sectional area } dx$$

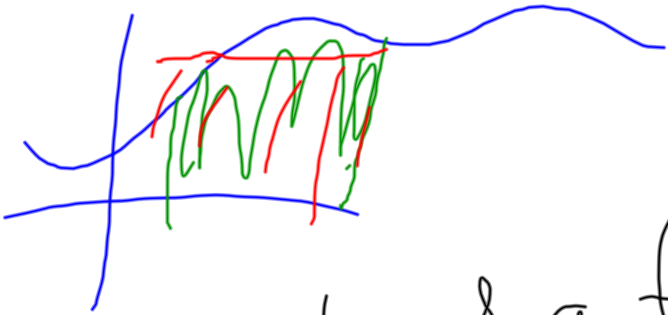
find rel min/max

- find critical numbers
  (x values where
  $f'(x) = 0$ or und)
- prove rel max or
  rel min
  → sign chart for
  sign of f'
  dec → inc
  OR
  → value of $f''(x)$

mean value theorem



mean value theorem for integrals



Avg value of a f^n

$$= \frac{1}{b-a}\int_a^b f(x)\,dx$$

related rates

- always true eqn
- always true deriv (wrt t)
- substitute

optimization

$\equiv$ rel min/max

Absolute max/min

rel max/min

+

Chk endpoints

Local Linear Approximation

$$f(x+h) \approx f(x) + f'(x)(h)$$

use to approximate a value of a f^n CLOSE to a known value

8.3 /26

$$\sum_{k=6}^{\infty} \frac{4}{\sqrt{k}}$$

Consider $\int_6^{\infty} \frac{4}{\sqrt{x}}\,dx$

$x^{-\frac{1}{2}}$

$$4 \lim_{b\to\infty} \int_6^b \frac{1}{\sqrt{x}}\,dx$$

$$= 4 \lim_{b\to\infty} 2x^{\frac{1}{2}} \Big|_6^b = 4 \lim_{b\to\infty} 2\sqrt{b} - 2\sqrt{6}$$

$= +\infty$

$\therefore$ series diverges

$$\lim \infty \cdot 0 \quad ?$$

2a

$$\sum k^{-11/10}$$

$$\int_1^{\infty} k^{-11/10}\,dk$$

$$= \lim_{b\to\infty} \left(\frac{k^{-1/10}}{-1/10} \right) \Big|_1^b$$

$$\lim_{b \to \infty} b^{?}$$

this limit diverges when

$\frac{1}{2}, \frac{1}{10}$ all positives

this limit converges when...

$-2, -\frac{11}{10}, -\frac{1}{10}$

all negatives

0

$$\int_1^\infty x^w \, dx$$

$$\lim_{b\to\infty} \int_1^b x^{w} \, dx$$

$$= \lim_{b\to\infty} \frac{x^{(w+1)}}{w+1}\Bigg|_1^b$$

$$= \frac{1}{w+1} \lim_{b\to\infty} b^{(w+1)} - 1^{w+1}$$

converges

$(w+1) < 0.$

so $w < -1$

except... $w \neq -1$

so $\int_1^\infty x^w \, dx$

converges when $w < -1$

OR

$\int_1^\infty \frac{1}{x^w} dx$ converges when $w > 1$

diverges when $w \leq 1$

$\therefore$

$\sum \frac{1}{k^w}$ converges when $w > 1$

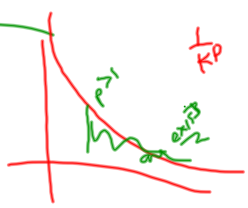
$1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$

$1 + 1 + 1 + 1$

$$\sum_{k=1}^\infty \frac{1}{k^p}$$

converges $p > 1$

diverges $p \leq 1$



$\frac{1}{k^p}$

$p = 1$ $\int_1^\infty \frac{1}{x} dx \to \infty$

$\lim_{b\to\infty} \ln b$

$\int_1^\infty \frac{1}{x^2} dx = \lim_{b\to\infty} -\frac{1}{x} \Big|_1^b = 1$

2b) $\sum_{6}^{\infty} \frac{1}{\sqrt{k}}$

compare

$\frac{1}{\sqrt{k}}$? $\frac{1}{k}$

$\sum_{k=1}^{\infty} \frac{1}{k}$

diverges
(by integral test)

HARMONIC

as $k \to \infty$
both $\to 0$

for $k > 1$

$k > \sqrt{k}$

so $\frac{1}{k} < \frac{1}{\sqrt{k}}$

Comparison Test

if $a_k \geq b_k$

and if $\sum_{k=1}^{\infty} b_k$ diverges

THEN $\sum_{k=1}^{\infty} a_k$ diverges

if $0 \leq a_k \leq b_k$

AND $\sum_{k=1}^{\infty} b_k$ CONVERGES

THEN $\sum_{k=1}^{\infty} a_k$ CONVERGES

8.3 a

$$\sum_{k=1}^{\infty} \frac{e^{1/k}}{k^2}$$

$$\int_1^{\infty} \frac{e^{1/x}}{x^2}\,dx$$

$u = \frac{1}{x}$

$du = -\frac{1}{x^2}\,dx$

$\cancel{x}$

Particle $\longrightarrow$ x-axis $\quad x(t) = \frac{1}{2}\sin(t) + \cos(2t)$.

$a\left(\frac{\pi}{2}\right) = ?$

$a(t) = v'(t) = x''(t)$

$x(t) = \frac{1}{2}\sin(t) + \cos(2t)$

$x'(t) = v(t) = \frac{1}{2}\cos(t) - 2\sin(2t)$

$a(t) = v'(t) = -\frac{1}{2}\sin(t) - 4\cos(2t)$

$a\left(\frac{\pi}{2}\right) = -\frac{1}{2}\sin\left(\frac{\pi}{2}\right) - 4\left(\cos\left(2\cdot\frac{\pi}{2}\right)\right)$

$= -\frac{1}{2}(1) - 4(\cos \pi) = -\frac{1}{2} - 4(-1)$

$= 3.5$

24) if $\boxed{\frac{dy}{dx} = y\cos x}$ & $y(0)=3$ then $y = ?$

$$\frac{dy}{dx} = y\cos x$$

$$\Downarrow$$

$$dy = y\cos x\,dx$$

$$\int \frac{1}{y}\,dy = \int \cos x\,dx$$

$$\ln|y| = \sin x + C$$

$\underline{y(0)=3}$

$\ln|3| = \sin(0) + C$

$\therefore C = \ln 3$

$\Rightarrow \ln|y| = \sin x + \ln 3$

$\Rightarrow e^{\ln y} = e^{(\sin x + \ln 3)}$

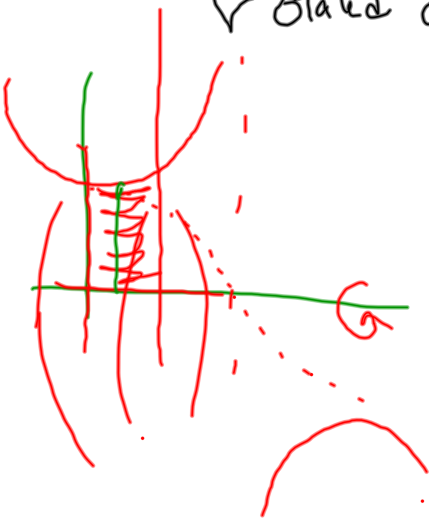
remove $|y|$ because we are int. in pt $(0,3)$ & near there y is always positive

$y = e^{\sin x} \cdot e^{\ln 3}$

$y = 3e^{\sin x}$

73/35) 1st quad bounded by $y = \sec x$, $x = \frac{\pi}{4}$, x-axis ; y-axis

rotated about ~~y~~ x-axis. Find vol



$$V = \int_0^{\frac{\pi}{4}} A(x)\,dx, \quad A(x) = \pi(\sec x)^2$$

$$= \pi \int_0^{\frac{\pi}{4}} \sec^2 x\,dx$$

$$= \pi (\tan x)\Big|_0^{\frac{\pi}{4}} =$$

$$= \pi\left(\tan\frac{\pi}{4} - \tan 0\right)$$

$$= \pi(1) = \pi$$

73/36 $y = e^{nx}$

find $\dfrac{d^n y}{dx^n} = n^n e^{nx}$

$y = e^{nx}$

$y' = ne^{nx}$

$y'' = n(ne^{nx}) = n^2 e^{nx}$

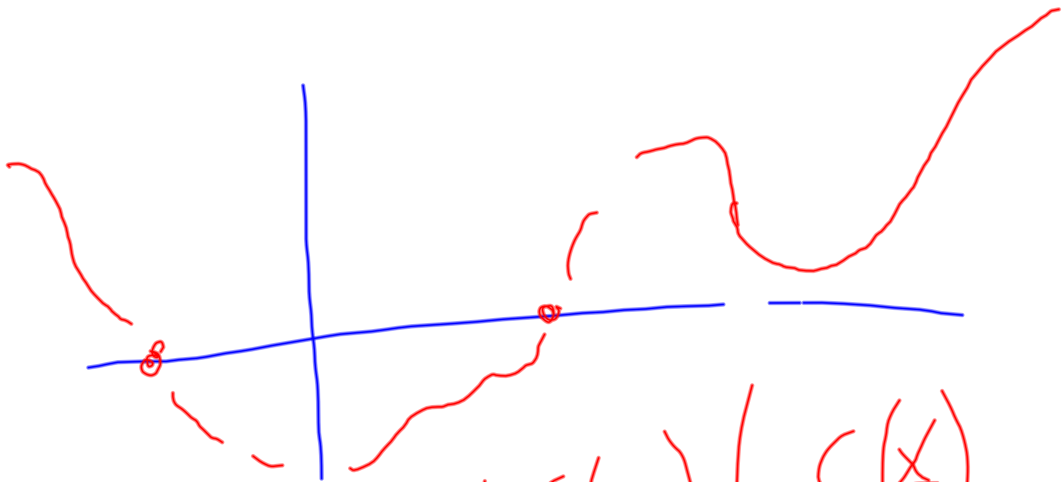
$y''' = n^2(ne^{nx}) = \underline{\underline{n^3 e^{nx}}}$

43) $\boxed{\text{sol'ns of } f(x)=0 \text{ are } x=-1,2.}$

~~sol'n~~

what are solutions of

$$f\left(\frac{x}{2}\right)=0$$



x	$f(x)$	$f\left(\frac{x}{2}\right)$
-1	0	$\frac{x}{2}=-1 \ \dots\ x=-2$
2	0	$\frac{x}{2}=2 \ \dots\ x=4$